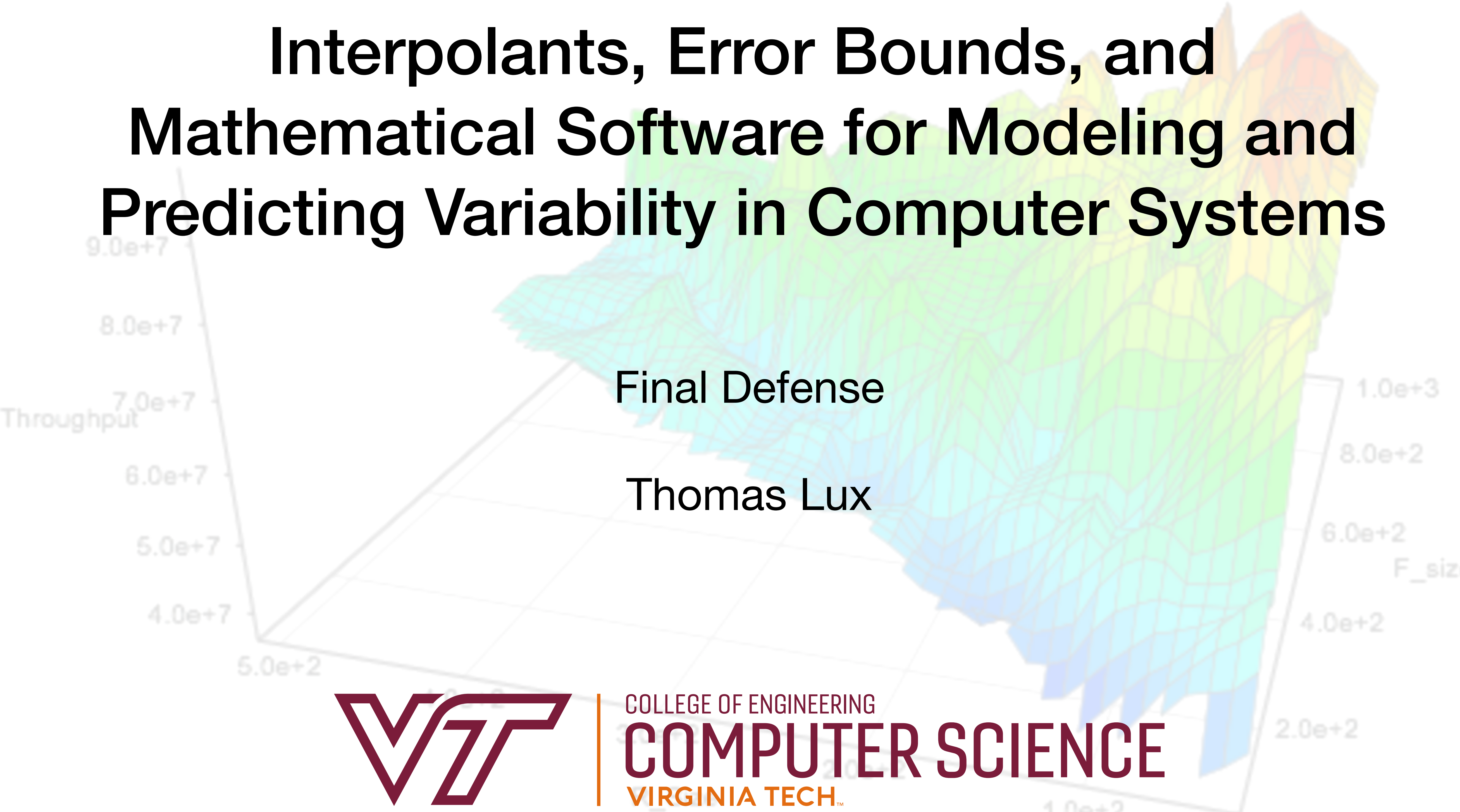
Interpolants, Error Bounds, and Mathematical Software for Modeling and Predicting Variability in Computer Systems

Final Defense

Thomas Lux



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Chapters

The Theory

$$\left|f(z) - \hat{f}(z)\right| \leq \frac{\gamma\|z - x_0\|_2^2}{2} + \frac{\sqrt{d}\,\gamma k^2}{2\sigma_d}\|z - x_0\|_2$$

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divided by how close the interpolated points are to being planar.

The Importance

The approximation error of a linear (simplicial) interpolant tends quadratically towards zero when approaching observed data only when the diameter of the simplex is also reduced proportionally.

In practice, only linear convergence to the true function can be achieved (because the evaluation points don't move).

Approximation error is largely determined by **data spacing**!

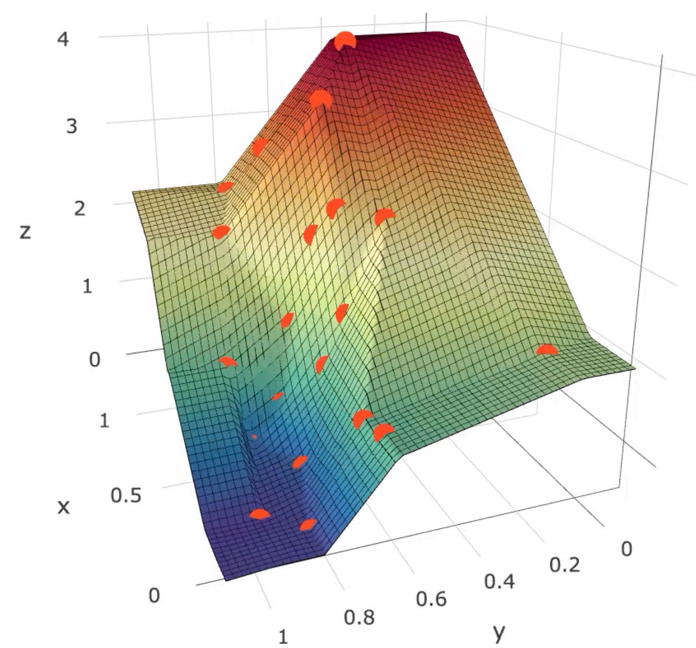
This theory only directly applies to Delaunay, but may give insight into the approximation behavior of other techniques.

Piecewise Linear Approximations

Delaunay
(interpolant)

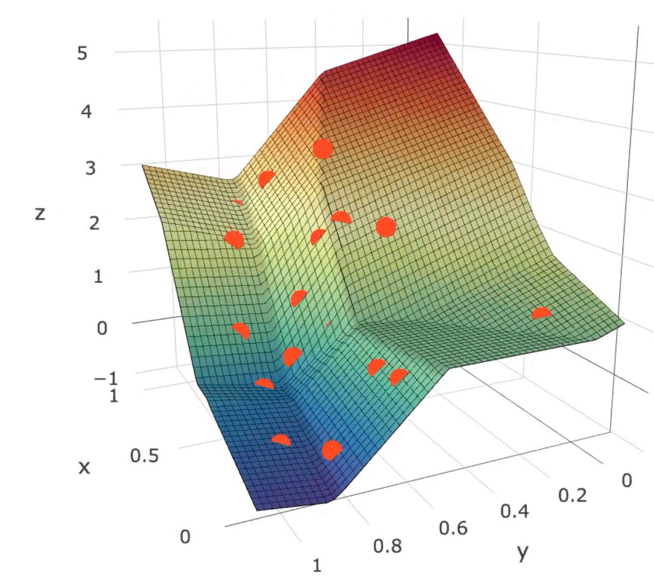
$$y = \sum_{i=0}^{d} w_i x^{(i)}, \quad \sum_{i=0}^{d} w_i = 1, \quad w_i \geq 0, \quad i = 0, \ldots, d$$

$$\hat{f}(y) = \sum_{i=0}^{d} w_i f\left(x^{(i)}\right)$$



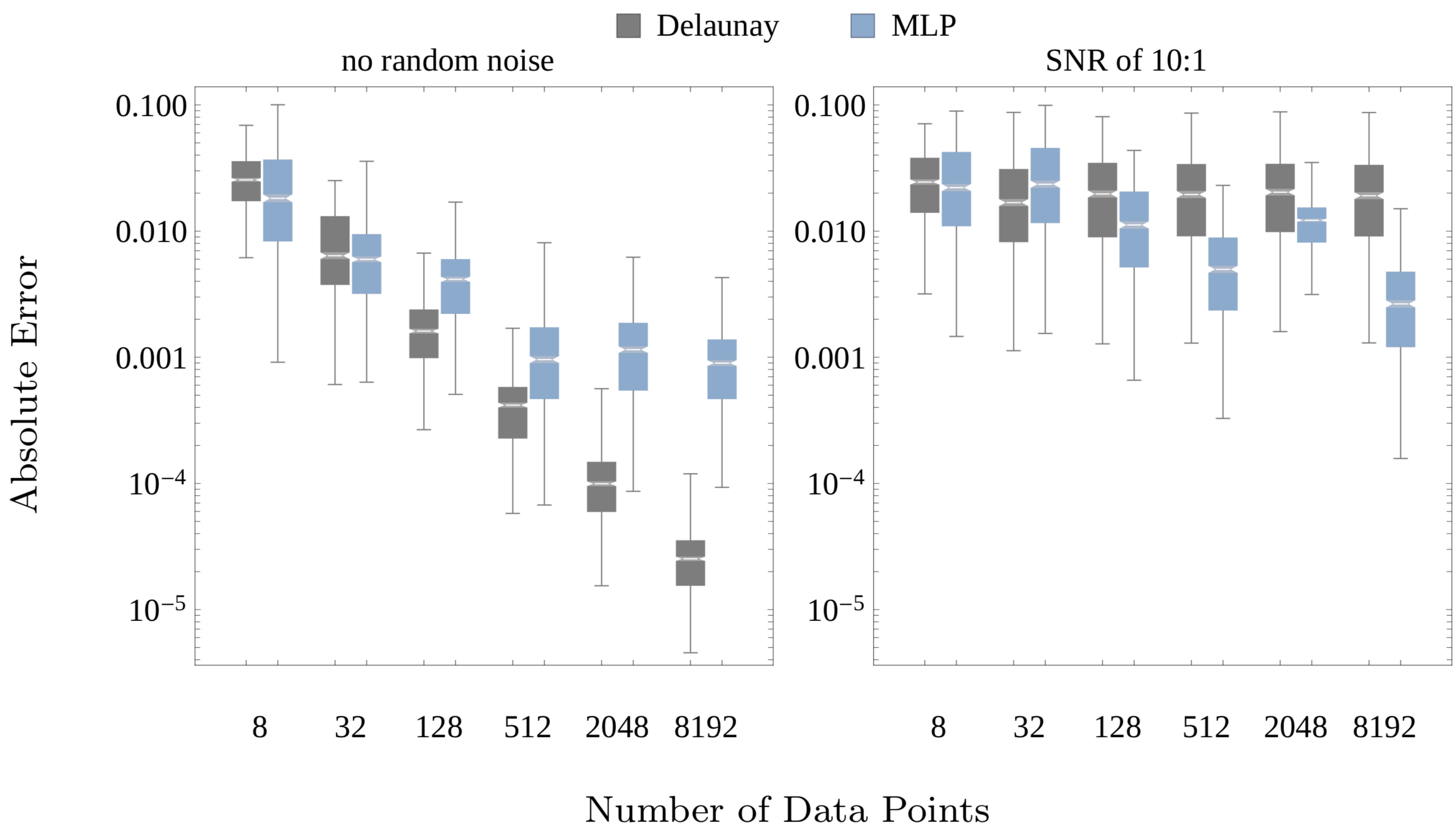
Multilayer Perceptron
(regressor)

$$l(u) = \left(u^t W_l + c_l\right)_+$$



Approximating $f(x) = \cos(\|x\|_2)$

In 2 dimensions, we get expected results.
Delaunay is better at interpolation, MLP better at regression.



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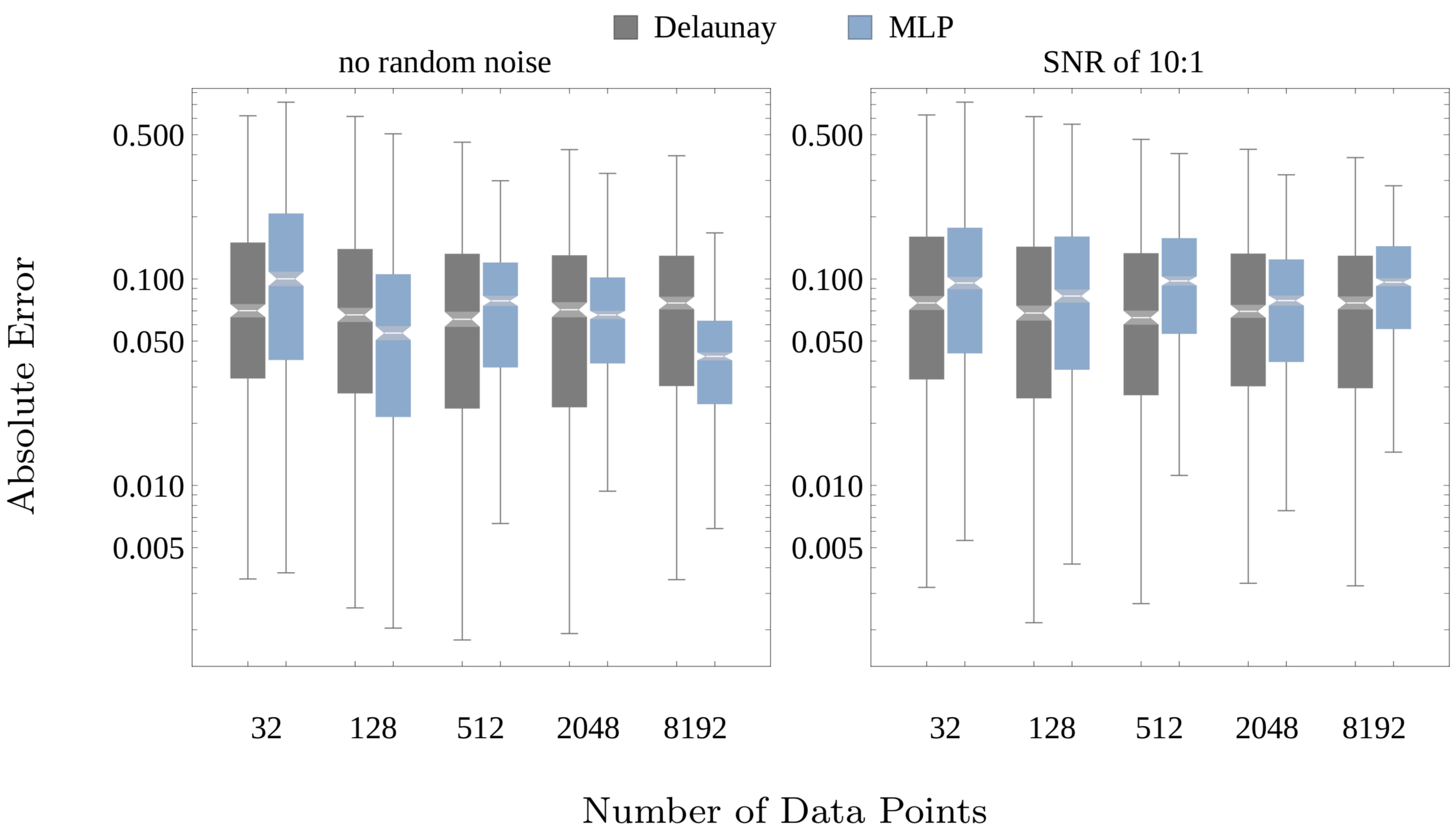
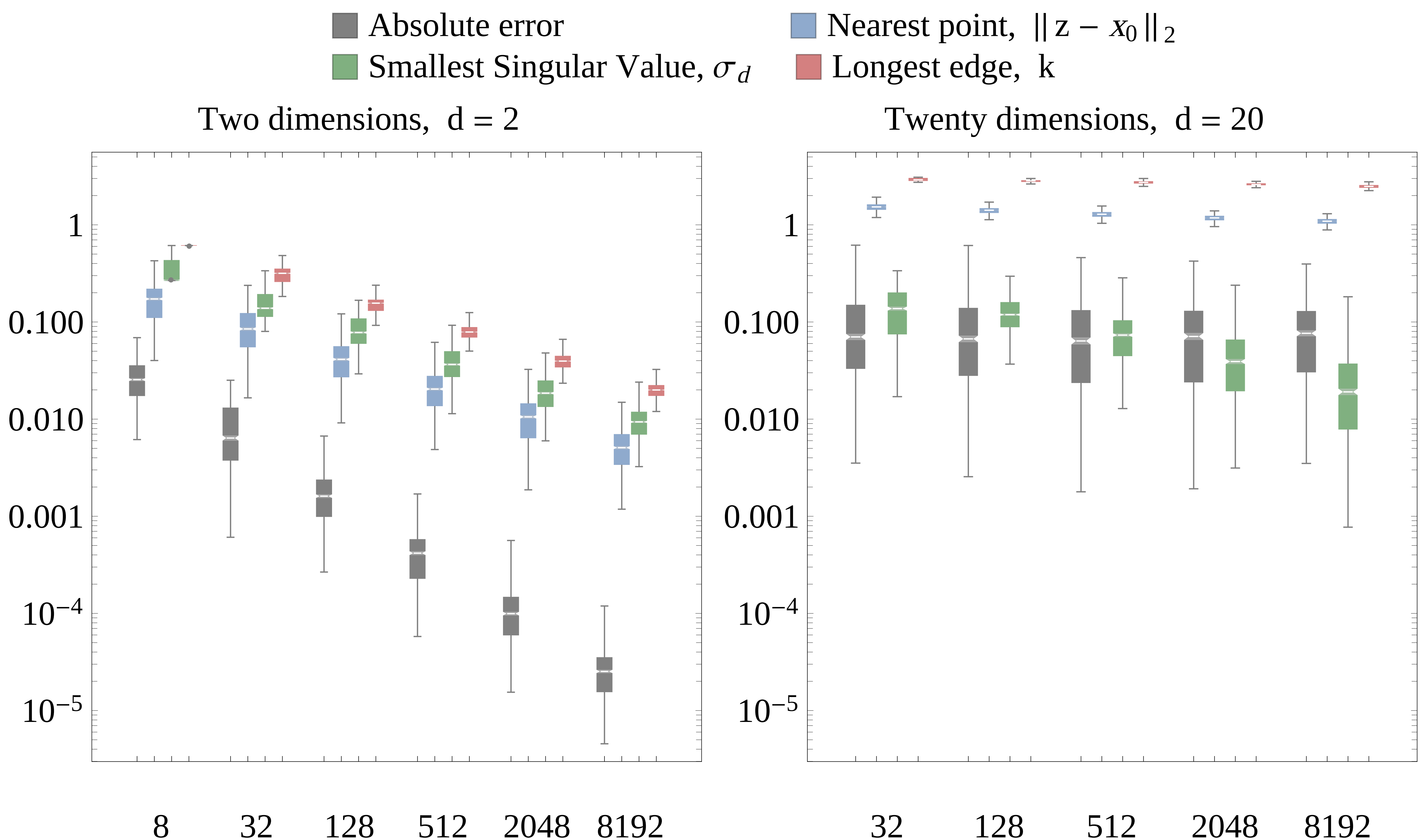
Approximating $f(x) = \cos(\|x\|_2)$

In 20 dimensions, the *intuitive* trend disappears!
Delaunay and MLP look the same.



Explaining the Convergence



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Connecting Back to Theory

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point doesn't get
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the longest edge does
not meaningfully shrink
in 20 dimensions with
thousands of points